SCALING EXPONENTS ESTIMATION FOR MULTISCALING PROCESSES

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ABSTRACT

We study the statistical performance of multiresolution (wavelet based) estimators commonly used for the estimation of the scaling exponents $\zeta(q)$ of multifractal processes. So far, such studies were conducted exclusively using the celebrated Mandelbrot's cascades. A new class of processes, compound Poisson cascades, with better statistical properties — stationary increments and continuous scale invariance — has recently been proposed in the literature. Making use of this new type of processes, we show that the multiresolution estimators are characterised by a generic and systematic feature: beyond a critical order q (which is determined analytically), they fail to estimate the $\zeta(q)$ and present instead a linear behaviour in q. We study in detail this linearisation effect and show that it does not disappear in the limit of infinite observation duration n and that the parameters characterising it do not depend on n. We comment on its major practical consequences and on its having been mostly overlooked in applications.

1. MOTIVATION

A relevant analysis of scaling phenomena and more precisely an accurate estimation of the corresponding scaling exponents $\zeta(q)$,

$$\mathbb{E}|X(t+a\tau_0)-X(t)|^q = c_q|a|^{\zeta(q)}, \qquad (1)$$

has appeared as a crucial issue in a wide variety of applications ranging from the study of the variability of body rhythms or hydrodynamic turbulence to internet data or stock market data modelling. Estimation of the $\zeta(q)$ is usually performed by estimating the moments of order q of some multiresolution quantities, such as wavelet coefficients, and tracking their power law behaviours as a function of the resolution (cf. Section 4). Besides self-similar ones [1], multifractal processes [2] constitute one of the major class of mathematical model used to account for scaling. However, the statistical performance of the estimators mentioned above when applied to multifractal processes received only little attention. A reason why may lie in the fact that the only theoretically known multifractal processes remained for a long time the celebrated Mandelbrot cascades [2, 3]. However, such processes have early been recognised to suffer from two major drawbacks: scale invariance is valid only for discrete scale ratios and does not hold continuously, the cascades do not form stationary processes. To overcome those difficulties, Barral and Mandelbrot recently proposed the construction of Multiplicative Products of Cylindrical Pulses, that will hereafter been referred to as Compound Poisson Cascades (CPC) [4]. The controlled scaling properties of these new processes, together with their *nicer* statistical properties, motivate and permit the present study of the statistical performance of the wavelet based estimators for the scaling exponents $\zeta(q)$ of multifractal processes. The major result reported in this work lies in the fact that the estimates $\hat{\zeta}(q,n)$ are undergoing a linearisation effect in their behaviour in q: for q below a critical value q_*, the $\hat{\zeta}(q,n)$ relevantly estimate the $\zeta(q)$ whereas for q above q_*, they necessarily behave linearly as a function of q. This is studied carefully and related to earlier work on close issues. This also sheds a new light on the analyses and uses of multifractal processes.

2. ESTIMATION

WAVELET COEFFICIENTS. Let ψ_0 denote a mother wavelet designed from a multiresolution analysis and characterised by its number of vanishing moments N. Let $\{\psi_{j,k}(t) = 2^{-j}\psi_0(2^{-j}t - k), (j,k) \in (\mathbf{Z}^+, \mathbf{Z})\}$ denote dilated and time-shifted templates of ψ_0 (note the unusual L_1 normalisation). Let $\{d_X(j,k) = \langle \psi_{j,k}, X\rangle$, $k \in \mathbf{Z}, j \in \mathbf{Z}^+\}$ stand for the Discrete Wavelet transform (DWT) coefficients of the process X to be analysed. The reader is referred to e.g., [5] for a detailed introduction to wavelet transforms.

STRUCTURE FUNCTIONS. Let us define the structure functions, as time averages of wavelet coefficients at a given scale 2^j:

$$S_n(q,j) = \frac{1}{n_j}\sum_{k=1}^{n_j} |d_X(j,k)|^q, \qquad (2)$$

where n denotes the observation duration of the process X (i.e., practically it means the samples $\{X(1),\dots,X(n)\}$ of X only are available) and n_j is the number of coefficients $d_X(j,k)$ at scale 2^j.

ESTIMATOR. The estimation consists in performing (non weighted) linear regressions in $j = \log_2 2^j$ versus $Y_{q,j} = \log_2 S_n(q,j)$ plots. This reads:

$$\hat{\zeta}(q,n) = \sum_{j=j_1}^{j_2} w_j Y_{q,j}, \qquad (3)$$

where the w_js simply read: $w_j = (S_0 j - S_1)/(S_0 S_2 - S_1^2)$, with $S_m = \sum_{j_1}^{j_2} j^m,\ m = 0,1,2$, and where the linear fit is performed over the continuous range of octaves $j \in [j_1, j_2]$.
By definition, wavelet coefficients are positive and negative random variables and for all the processes studied here have probability density functions that are continuous and strictly positive in 0: hence, their moments are defined only when $q > -1$ and so is the estimator $\hat{\zeta}(q,n)$.

3. COMPOUND POISSON CASCADES AND FRACTIONAL BROWNIAN MOTION IN MULTIFRACTAL TIME

COMPOUND POISSON CASCADES. Up to our knowledge, the only stochastic processes characterised by scaling behaviours of the type sketched in Eq. (1) and that can be a priori prescribed are based on multiplicative cascade constructions. For a long time, the only such known and widely used processes were the celebrated Mandelbrot's cascades Q_r [3]. However, this is also well-known that such processes suffer from two major drawbacks when aiming at real data modelling: they posses only discrete scale invariance (i.e., the scaling in Eq. (1) are valid only for a discrete subset of scale ratios, e.g., 2^j for recursive binary splitting), Q_r do not form stationary processes. This results from the recursive binary splitting construction that is based on a rigid time-scale geometry.
To overcome such drawbacks, a new process has very recently been introduced and studied by Barral & Mandelbrot [4]. This construction starts with a random point process $(t_i, r_i)_{i\in\mathcal{I}}$, defined on a rectangle $I = \{(t', r') : r \le r' \le 1, -1/2 \le t' \le T + 1/2\}$ and with density $dm(t, r)$. To the $(t_i, r_i)_{i\in\mathcal{I}}$ are associated positive i.i.d. multipliers W_i, with mean one. The corresponding density $Q_r(t)$, referred to as compound Poisson cascade (CPC, hereafter), is then defined as the product of the W_i corresponding to points within the cone $\mathcal{C}_r(t) = \{(t', r') : r \le r' \le 1, t - r'/2 \le t' \le t + r'/2\}$ (where the normalisation factor ensures $\mathbb{E}Q_r = 1$):

$$Q_r(t) = \left(\mathbb{E}[\prod_{(t_i,r_i)\in\mathcal{C}_r(t)} W_i]\right)^{-1} \prod_{(t_i,r_i)\in\mathcal{C}_r(t)} W_i. \quad (4)$$

FRACTIONAL BROWNIAN MOTION IN MULTIFRACTAL TIME. To turn this cascade Q_r into a process with positive and negative fluctuations, one can follow an idea which goes back to Mandelbrot [6]. It will be referred hereafter as the *Fractional Brownian motion in Multifractal time based on a CPC*, CPC-MTFBM, and is defined as follows. Let $A = \lim_{r\to 0} \int_0^t Q_r(s)ds$ denote the measure obtained from the CPC density Q_r and let B_H stand for fractional Brownian motion with Hurst parameter H [1]. The CPC-MTFBM is defined as:

$$V_H(t) = B_H(A(t)), \quad t \in \mathrm{R}^+. \quad (5)$$

STATIONARY INCREMENTS AND SCALING BEHAVIOURS. If the density $dm(t, r)$ is chosen to be covariant under time translation, $dm(t, r) = g(r)drdt$, the $Q_r(t)$ are stationary processes, while $A(t)$ and $V_H(t)$ have stationary increments. Moreover, if $dm(t, r)$ is chosen such that the average number of points on $(t_i, r_i)_{i\in\mathcal{I}}$ is proportional to $-\log r$, i.e., if $dm(t, r) = c(1/r^2 + \delta(1 - r))drdt$, then $Q_r(t)$, $A(t)$ and $V_H(t)$ present power law behaviours as in Eq. (1) that holds continuously for all dilation factors a. The corresponding proofs can be found in [4] for Q_r and A, and in [7, 8] for V_H. The scaling read respectively:

$$\left.\begin{array}{lcl} \mathbb{E}(\frac{1}{a\tau_0}\int_t^{t+a\tau_0} Q_r(u)du)^q & = & c_q|a|^{\varphi(q)}, \\ \mathbb{E}|A(t+a\tau_0) - A(t)|^q & = & c'_q|a|^{q+\varphi(q)}, \\ \mathbb{E}|V_H(t+a\tau_0) - V_H(t)|^q & = & c''_q|a|^{qH+\varphi(qH)}, \end{array}\right\} \quad (6)$$

where $\varphi(q) = c((1 - \mathbb{E}W^q) - q(1 - EW))$. Here, we focus our interest on the (estimation of the) $\zeta(q) = qH + \varphi(qH)$ of V_H.

DEPTH OF THE CASCADE AND OBSERVATION DURATION. We have developed the procedures used to numerically synthetize CPC-MTFBM in MATLAB. They are documented in [7]. The practical synthesis of CPC-MTFBM implies that scaling behaviours hold within a minimal and a maximal (or integral) scales (as can be seen in the definition of the cone above through $r \le r' \le 1$). In the present work, the corresponding scales are, by convention, set to 2^0 and and 2^J, respectively. The total number of samples of V_H is chosen as $n = 2^{J_L}$ so that the relevant parameter controlling the observation duration has to be written in numbers of integral scales: $2^{J_L}/2^J$.

CRITICAL POINTS AND LEGENDRE TRANSFORM. For reasons made clear in the next section, we define the Legendre transform $D(h) = 1 + \min_q\ (qh - \zeta(q))$ of the 1D function $\zeta(q)$ and the theoretical critical points, $D_*^\pm, h_*^\pm, q_*^\pm$, as:

$$\left.\begin{array}{lll} (h_*^\pm, D_*^\pm) & \text{such that} & D_*^\pm = 0 \quad \text{and } D(h_*^\pm) = 0, \\ q_*^\pm & \text{such that} & h_*^\pm = \quad (d\zeta(q)/dq)_{q=q_*^\pm}. \end{array}\right\} \quad (7)$$

For simplicity, we will only consider here processes such that $q_*^- < -1$ and will therefore further use q_*^+, h_*^+, D_*^+ only.

4. LINEARISATION EFFECT

METHODOLOGY. The performance of $\hat{\zeta}(q, n)$ are obtained from numerical simulations: $nbreal$ copies of CPC-MTFBM, with $\mathbb{E}W^q = c(1 - \exp(\mu q + \sigma q^2))$, are produced and the estimators $\hat{\zeta}(q, n)$ are applied to each of them. Bias, variance and statistical behaviour of the $\hat{\zeta}(q, n)$ are deduced from averaging over realisations. In the present work, we selected: $N = 3$, $nbreal = 1000$, $J = 11$, $j_1 = 3$, $j_2 = 9$, $n = 2^{J_L}$ with $J_L = 9, \ldots, 16$.

LINEARISATION EFFECT. On all the simulations conducted, we observed the following fundamental fact: the wavelet based estimators for the $\zeta(q)$ are undergoing a *linearisation effect* as a function of q. While q belongs to a specific interval $q \in [-1, \hat{q}_o]$, the estimate $\hat{\zeta}(q, n)$ account for the theoretical $\zeta(q)$, as defined in Section 3. For q outside this interval, $q \ge \hat{q}_o$, the $\hat{\zeta}(q, n)$ significantly depart from the $\zeta(q)$ and, besides that, the $\hat{\zeta}(q, n)$ necessarily behave as a linear function of q, $\hat{\zeta}(q, n) = \hat{\alpha}_o + \hat{\beta}_o q$, for each and every replication of the process. This is illustrated in Fig. 1, on a single (top left) and on ten (bottom left) replications. Moreover, it can be seen that $\hat{\alpha}_o$ and $\hat{\beta}_o$ are random variables that depend on each replication.

LEGENDRE TRANSFORM. To further study this linearisation effect, let us compare the Legendre transform $\hat{D}(h, n)$ of the $\hat{\zeta}(q, n)$ to that $D(h)$ of the theoretical function $\zeta(q)$. Fig. 1, top right, shows that, for $h \ge \hat{h}_o$, $\hat{D}(h, n)$ tends to (superimpose to) $D(h)$. Fig. 1 shows as well that $\hat{D}(h, n)$ is abruptly ended by an *accumulation point*, with coordinates denoted by $(\hat{h}_o, \hat{D}_o)$, which constitutes another evidence for and signature of the linearisation effect. Fig. 1, bottom right, shows how accumulation points $(\hat{h}_o, \hat{D}_o)$, obtained from hundreds of replications of the same process, spread in the neighbourhood of the theoretical curve $D(h)$ and mainly concentrate around the critical point: $(h_*^+, D_*^+ = 0)$.

DEPENDENCE ON THE OBSERVATION DURATION n. Fig. 2 shows the behaviours of the parameters $\hat{\alpha}_o$, $\hat{\beta}_o$, $\hat{D}_o$, $\hat{h}_o$ and $\hat{q}_o$, defining the linearisation effect, as a function of the observation duration n. The value $\hat{q}_o$ is obtained straightforwardly by comparing (and equating) $|\hat{\zeta}(q,n) - \zeta(q)|$ and $|\hat{\zeta}(q,n) - (\hat{\alpha}_o + \hat{\beta}_o q)|$. Striking conclusions can be inferred from Fig. 2 (left column). First, the linearisation effect does not disappear when the observation duration increases, $n \to +\infty$: this is not a finite size effect. Second, the average values of the parameters characterising it do not depend on n: the average critical q above which the linearisation occur does not vary with n, nor does the observed average affine function $\alpha + \beta q$. Third, the variances of the fluctuations of the parameters characterising the linearisation effect decrease as long as the observation duration n is shorter than the integral scale, $n \le 2^J$, but remain constant as soon as $n \ge 2^J$ ($2^J = 2^{11}$ in Fig. 2). This implies that the amplitude of the statistical fluctuations of the parameters characterising the linearisation effect do not decrease while the observation duration increases.

CONJECTURE. The empirical observation described above, obtained on a large number of numerical simulations as well as for a large variety of choices for $\varphi(q)$, N and ψ_0, leads us to formulate the following conjecture regarding the behaviour of the $\hat{\zeta}(q,n)$:

$$\begin{array}{rclll} & \hat{\zeta}(q,n) & \to & \zeta(q), & -1 < q \le q_*^+, \\ \hat{\zeta}(q,n) & = \hat{\alpha}_o + \hat{\beta}_o q & \to & 1 - D_*^+ + h_*^+ q, & q_*^+ \le q. \end{array} \tag{8}$$

where q_*^+, h_*^+, D_*^+ are defined in Eq. (7) and where $X \to x$ does not stand for asymptotic convergence when $n \to +\infty$ but simply indicates that X is a random variable spread around the deterministic quantity x. Let us put the emphasis moreover on the fact that, for $q \ge q_*^+$, the $\hat{\zeta}(q,n)$ behave linearly for each replication of the process and not only on average.

COMMENTS AND INTERPRETATION. An equivalent result had been obtained in [9] for the Mandelbrot's cascades and the box-aggregation estimator. This had also been studied for those cases in [10, 11, 12] and for wavelets and conservative Mandelbrot's cascades in [13]. Our results are in agreement with those reported in the above mentioned papers and involve the same critical parameters (cf. Eq. (7)). They extend them to CPC cascades, MTFBM processes and wavelet based estimators, for which, up to our knowledge, this linearisation effect had never been studied. A reason why this extension holds for CPC may lie in their being multiplicative martingales [4], as are the original Mandelbrot's cascades.
Other studies of the linearisation effect related to Mandelbrot's cascades and box aggregration estimator are available in the literature (see e.g., [14, 15, 16]). They associate it to finite size effects , maximal observable singularity or finiteness of moment of the cascades Q_r. So far, our observations and results significantly departs from theirs (cf. [17]).
This linearisation effect, whose evidence and characterisation we see as the major result of the present work, implies a fundamental practical consequence: the analysis of multifractal processes should no longer consist in estimating the scaling exponents $\zeta(q)$ for all qs but rather in estimating the parameters of the linearisation effect q_*^+, h_*^+, D_*^+ and then in estimating the $\zeta(q)$ for $q \le q_*^+$.

5. ESTIMATION OF THE CRITICAL POINTS

PROCEDURES. We now propose estimators for the parameters q_*^+, h_*^+, D_*^+ from a single observation of the process CPC-MTFBM. Based on the observation that the fluctuations of the critical parameters do not decrease when n increases, the observation X of length n is first splitted into L blocks $X_l, l = 1, \ldots, L$, of sizes 2^{J_L+2}. On each block, the following estimation procedure is applied. For $q \in (-1, \ldots, q_M]$ with $q_M \gg q_*^+$, estimate $\hat{\zeta}_l(q,n)$ as in Eq. (3). For the corresponding Legendre transform $\hat{D}_l(h,n)$, perform a second order polynomial fit, $\tilde{D}_l(h,n)$, within a domain selected through $h \le h_R$ and $\hat{D}_{l,n} \ge D_R$ and compute the inverse Legendre transform $\tilde{\zeta}_l(q,n)$. Using $\tilde{d}_{X_l}(j,k;q_R) = |d_{X_l}(j,k)|^{q_R}/S_n(q_R,j)$ (following [18]), define $\hat{h}_{l,*}^+, \hat{D}_{l,*}^+$ as:

$$\begin{array}{rcl} \hat{h}_{l,*}^+ & = & \sum_j w_j \sum_k \tilde{d}_{X_l}(j,k;q_R).\log_2 |d_{X_l}(j,k)|, \\ \hat{D}_{l,*}^+ & = & \sum_j w_j \sum_k \tilde{d}_{X_l}(j,k;q_R).\log_2 (\tilde{d}_{X_l}(j,k;q_R)). \end{array} \tag{9}$$

Define $\hat{q}_{l,*}^+$ as the value of q such that $|\hat{\zeta}_l(q,n) - (1 - \hat{D}_{l,*}^+ + q\hat{h}_{l,*}^+)| = |\hat{\zeta}_l(q,n) - \tilde{\zeta}_l(q,n)|$. Define $\hat{h}_*^+$, $\hat{D}_*^+$ and $\hat{q}_*^+$ as the averages of the corresponding $\hat{h}_{l,*}^+$, $\hat{D}_{l,*}^+$ and $\hat{q}_{l,*}^+$ over the L blocks. In this work, the parameters are chosen as $h_R = h$ such that $\hat{D}_l(h,n)$ is maximal, $D_R = 0.3$, $q_R = q_M \simeq 3q_*^+$.

RESULTS. Results reported here are obtained from numerical simulations run over $nbreal$ replications of the same process with observation duration $n = L.2^{11}$, $2 \le L \le 2^5$. Fig. 3 clearly indicates that, despite their being preliminary and elementary, $\hat{q}_*^+$, $\hat{h}_*^+$ and $\hat{D}_*^+$ provide us with relevant estimates of satisfactory orders of magnitude for the parameters characterising the linear effect. Their variances decrease as n^{-1}. However, they present a systematic residual bias that does not decrease while n increases. This is under current study and consistent with finding in [19]. Automatic selection of the parameters h_R, D_R, q_R, q_M is under study. Up to our knowledge, they are the first estimators proposed to measure the parameters defining the linearisation effect and are working with a single finite length observation of the analysed process

6. CONCLUSION

Making use of CPC and MTFBM, we showed that scaling exponents $\zeta(q)$ for multifractal processes are meaningful quantities only for a finite range of values of qs and that the corresponding wavelet based estimators undergo a linear behaviour outside this range. It has been checked elsewhere that this is a generic effect observed for all multifractal processes and types of cascades so far proposed in the literature and for all multiresolution based estimators (box-aggregation, increments, wavelets) [17]. This extends earlier findings on Mandelbrot's cascades and box-aggregation estimators (cf. [9]). A similar linearisation effect occurs around the critical point q_*^-, h_*^-, D_*^-, though more intricate to study.
Despite its systematic nature, this effect has been widely overlooked in applications and in the actual analysis of empirical time series. However, it may play a key role in the tasks of distinguishing between various multifractal models or discriminating between monofractality or multifractality. Its impact on the analysis of data from hydrodynamic turbulence and internet network traffic is under current investigation.

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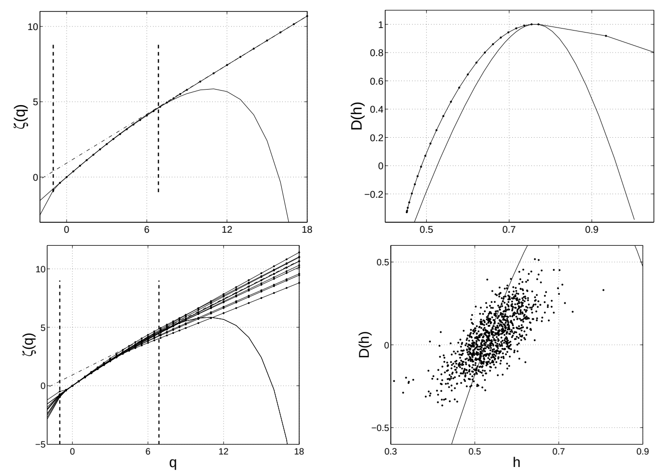

Fig. 1. **Linearisation Effect:** Theoretical and estimated $\zeta(q)$ (left column) and Legendre Transforms $D(h)$ (right column) on a single replication (first raw), on 10 replications (second raw).

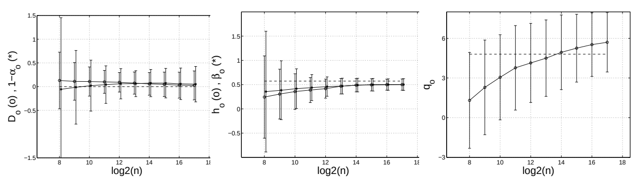

Fig. 2. **Dependence on the observation duration:** Means and variances of the parameters characterising the linearisation effect (left, $\hat{D}_o, 1-\hat{\alpha}_o$, middle, $\hat{h}_o, \hat{\beta}_o$, right $\hat{q}_o$) as a function of $\log_2(n)$ with respects to the critical values D_*^+, h_*^+, q_*^+, the integral scale corresponds to 2^{11} samples.

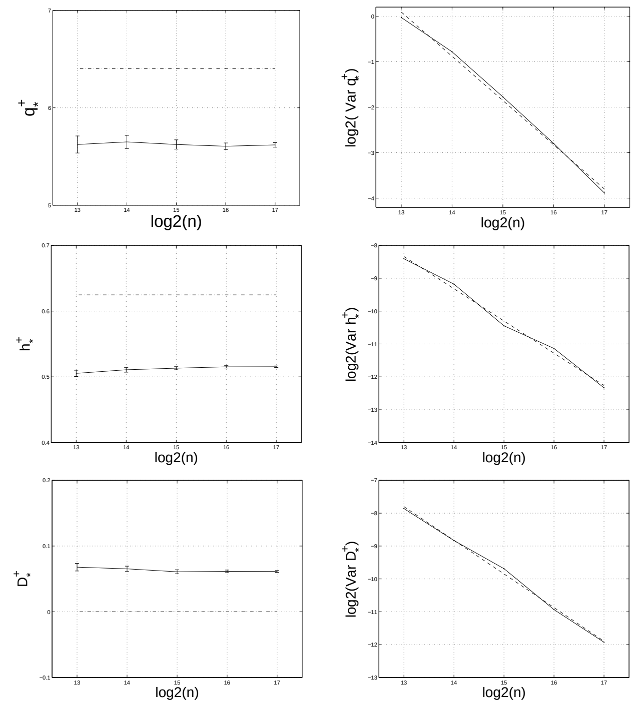

Fig. 3. **Estimation of the critical points:** From top to bottom, $\hat{q}_*^+, \hat{h}_*^+, \hat{D}_*^+$, Means and (log) Variances, left and right columns.

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