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ACOUSTIC TOMOGRAPHY

Ultrasonic Tomography of Nonmixing Fluid Flows

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Abstract—An ultrasonic method for simultaneous measurement of two-dimensional distributions of compositions and flow velocities in a system of two nonmixing fluids is proposed and implemented. The method is based on tomographic reconstruction of images of scalar and vector objects using a rectangular system of stationary transceivers. Images of simple objects (a vortex in a homogeneous fluid and a standing gravitational-capillary wave in a system of two nonmixing fluids) are obtained by this method.

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1. INTRODUCTION

Currently, the technique of tomographic reconstruction is widely used in medical diagnostics, nondestructive examination, and introscopy systems [1]. Despite the fact that the methods of X-ray and magnetic resonance tomographies have become most popular, a number of specific features of ultrasonic analysis allow one to gain additional data on the properties of objects under study [2].

The measurement parameter in ultrasonic tomography is generally taken to be the propagation time of a probe signal through a medium. For fluids, the difference in the signal delay times in the forward and backward directions contains information about the longitudinal flow velocity, while the sum of delays characterizes the speed of sound in the propagation medium. Data on the speed of sound make it possible to find the concentration distributions in mixed media [3] or temperature distributions in various objects [4]. It was shown in [5] that the sensitivity of the difference in ultrasonic signal propagation times in opposite directions in a medium to a change in the speed of sound exceeds many times the sensitivity of the sum of propagation times to this change. At the same time, the sensitivities of the difference and sum of propagation times to a change in the flow velocity are opposite. This circumstance allows one to separate algorithms for determining these parameters [6, 7].

In this paper, we report the results of studying the stationary parametric oscillations in a system of two nonmixing fluids by ultrasonic tomography. The generation of oscillations under conditions of Faraday instability [8] is used, for example, for produce mixtures of two liquids in a closed volume [9]. Currently, these processes are observed mainly by means of optical detection [9]; as a result, the analysis is generally restricted to the behavior of interfaces between optically transparent media. At the same time, data on the flows formed in these fluids are of undoubted interest.

The use of time-sensitive ultrasonic methods makes it possible to record perturbations throughout the entire volume of a medium both due to its motion with respect to the receiver and transmitter and due

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to the change in its composition (and, correspondingly, the speed of sound). Thus, the solution of a tomographic problem allows one to obtain separately a scalar field of composition distribution and a vector field of velocities of dissimilar media.

2. BASIC EQUATIONS OF ULTRASONIC TOMOGRAPHY

The propagation path of an acoustic wave in a moving dispersed medium (see the schematic diagram in Fig. 1) is determined by two factors: the refraction of rays in regions with different speeds of sound **c** and the path displacement related to the motion of the medium with a velocity **u**. In this case, the group velocity is $\mathbf{v}=\mathbf{c}+\mathbf{u}$, where $\mathbf{c}=c\mathbf{k}/k$ is the speed of sound and **k** is the wavevector. The components of the velocity of the medium are $u_x=u\sin\alpha$ and $u_y=-u\cos\alpha$, where angle α determines the flow velocity direction, θ is the angle between wavevector **k** and the tangent to the ray path, and φ is the angle of ray rotation. The ultrasonic wave propagation time from transmitter i to receiver j and in the backward direction depends on the tangential velocity component $v_l=c\cos\theta\pm u\sin(\alpha+\varphi)$ and is given by the formula

$$t^{\pm}_{i,j}=\int_0^{L_{i,j}}\frac{dl}{c\cos\theta\pm u\sin(\alpha+\varphi)}. \qquad (1)$$

The difference in the forward and backward propagation times between transmitters i and j, with allowance for the small value of flow velocity ($u\ll c$), is

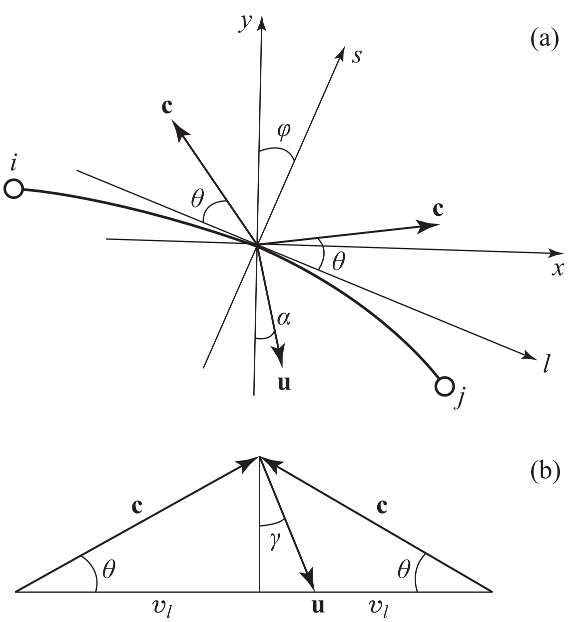

Fig. 1. Ultrasonic tomography of moving objects: (a) an ultrasonic ray path and (b) a velocity vector diagram.

$$\Delta t_{i,j}=2\int_0^{L_{i,j}}\frac{u}{c^2}\sin(\alpha+\varphi)\,dl, \qquad (2)$$

where $\Delta t_{i,j}=t_{i,j}-t_{j,i}$ and $L_{i,j}$ is the path length between the ultrasonic wave transmitter and receiver. According to Fermat's principle, the sound ray path should provide a minimum propagation time between two points. This minimum is yielded by the maximum value of the tangential velocity v_l; hence, the minimum value of the normal velocity throughout the entire wave propagation path should be zero. Thus, the following equation can be written for the angle of displacement θ:

$$c\sin\theta=-u\cos(\alpha+\varphi). \qquad (3)$$

Having taken into account Eq. (3), one can separate the flow velocity components that are necessary to reconstruct the vector velocity field:

$$\begin{aligned}\int_0^{L_{i,j}}\frac{u_x}{c^2}dl&=\frac{\Delta t_{i,j}}{2}\cos\varphi+\int_0^{L_{i,j}}\frac{\sin\theta\sin\varphi}{c}dl,\\ \int_0^{L_{i,j}}\frac{u_y}{c^2}dl&=-\frac{\Delta t_{i,j}}{2}\sin\varphi+\int_0^{L_{i,j}}\frac{\sin\theta\cos\varphi}{c}dl.\end{aligned} \qquad (4)$$

When studying the motion of fluids in closed volumes, where only vortex flows arise ($\operatorname{div}\mathbf{u}=0$), system of equations (4) is simplified because the last terms on the right-hand side of the equations are zero.

The composition (or concentration) distribution in a system (mixture) of two fluids can be estimated from the distribution of the speed of sound, the data on which are present, for example, in the sum of ultrasonic wave propagation times in the forward and backward directions. Thus, we obtain a closed system of independent integral equations, which allows one to reconstruct the concentration distribution and vector field of fluid flow velocities:

$$\begin{aligned}\int_0^{L_{i,j}}\frac{u_x}{c^2}dl&=\frac{\Delta t_{i,j}}{2}\cos\varphi,\\ \int_0^{L_{i,j}}\frac{u_y}{c^2}dl&=-\frac{\Delta t_{i,j}}{2}\sin\varphi,\\ \int_0^{L_{i,j}}\frac{dl}{c}&=\frac{\sigma t_{i,j}}{2},\end{aligned} \qquad (5)$$

where $\sigma t_{i,j}=t_{i,j}+t_{j,i}$.

The equations in basic system (5) are Radon equations, the solution algorithm of which is widely used in tomographic reconstruction systems. The back projection method is generally used in conventional tomography to reconstruct object images. In this case, the Radon integral is reduced to the convolution equation, the solution of which is transferred to Fourier space. A specific feature of this approach is the determination of an optimal filter, which is necessary to reconstruct correctly the object image [10]. For this reason, the method is referred to as a filtered back projection (FBP) method. However, the algorithm of search for an optimal filter is fairly complicated [11]; it has been thoroughly developed for only tomographic systems with conventional circular scanning.

Here, the algebraic reconstruction method was used to reconstruct the vector field of flow velocities. In this case, the matrix of ray time delays in the scanning region is determined by the relation

$$T_{i,j} = \sum_{m,n} A^{i,j}_{m,n} O_{m,n}, \tag{6}$$

where $T_{i,j}$ is the time-delay matrix, $A^{i,j}_{m,n}$ is the ray tensor, and $O_{m,n}$ is the object matrix; the ranges of variation in indices are $i=[1,I]$, $j=[1,J]$, $m=[1,M]$, and $n=[1,N]$. After reducing the matrices to the vector form, Eq. (6) can be rewritten as a matrix equation

$$A_{p,q}\,\mathbf{o}_p = \mathbf{t}_q, \tag{7}$$

where $p=[1, M\times N]$ is a pixel number in the image, $q=[1, I\times J]$ is an acoustical ray number.

To solve matrix equation (7), one must multiply both parts of (7) by transposed matrix A^T. In addition, it is necessary to take into account that A^TA matrix is degenerated in almost all cases. We sought for a solution using the Tikhonov regularization [12]; i.e., the solution was sought for in the form

$$\mathbf{o} = (A^TA + \delta E)^{-1} A^T \mathbf{t}, \tag{8}$$

where δ is the regularization parameter and E is the identity matrix.

3. EXPERIMENTAL SETUP AND MEASUREMENT TECHNIQUE

The experiment geometry was chosen in correspondence with the conditions for laboratory observations of parametric Faraday instability [9].

Experiments were performed in an optically transparent cell 50×50×25 mm in size (see the photograph in Fig. 2). Four 16-element ultrasonic longitudinal wave arrays (ACS Ltd) with a central operating frequency of 4 MHz were inserted in the cell lateral walls. The array working area was 39×10 mm^2; thus,

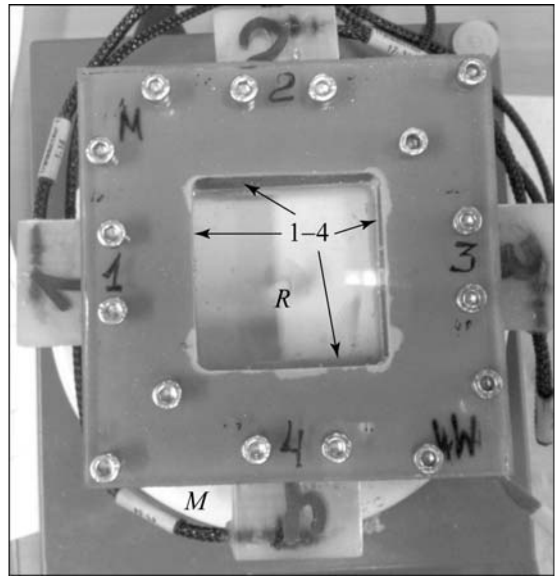

Fig. 2. Experimental cell: (1–4) working surfaces of ultrasonic arrays, M is the magnetic mixer platform, and R is the magnetic mixer rotor.

most of the cell perimeter was occupied by a multielement transceiving acoustic system. Ultrasound was excited and received using a 64-channel MultiX 2000 scanning system (M2M Ltd), which made it possible to implement a tomographic scheme with a stationary transceiving elements.

The operation algorithm implied alternate pulsed excitation (pulse width 100 ns) of each out of 64 piezoelectric elements located along the cell perimeter and recording of the wave forms received by all other elements in each excitation cycle. In the ray approximation, this is equivalent to dealing with a set of 64 rays propagating through the cell; the system retains 64×64 = 4096 signals for the complete cycle. The maximum pulse repetition rate (or transmitter successive switching rate) depended (as in ultrasonic scanner) impartially on such factors as the geometric sizes of the cell and the speed of sound in the fluids filling it. A subjective limitation was also imposed by the operating speed of the data transfer channel in the scanning system. In the experiments described here, the repetition rate was 800 Hz, which determined the total circular path time for all transmitters to be 80 ms.

The measured parameter for each received signal (waveform) was the delay time. To determine this characteristic, we performed preliminary reference measurements of all waveforms for a cell filled with stationary water. Then the change in the delay time was determined from the point corresponding to the maximum of correlation function for each waveform and the corresponding reference form. The thus formed delay variation matrix T had a dimension of

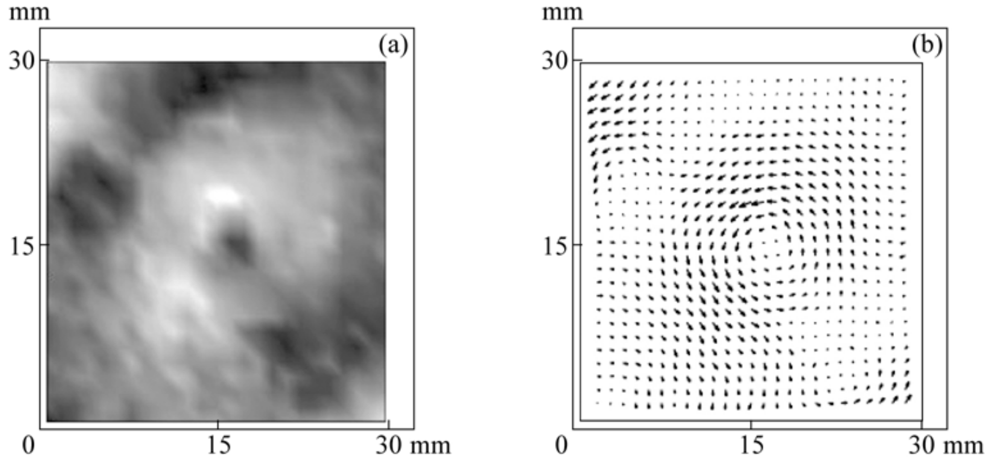

Fig. 3. Tomographic reconstruction of flow velocity in a homogeneous fluid: (a) velocity modulus and (b) velocity vector field.

64×64 and served as an initial data set for reconstruction. The elements of matrix T corresponding to transmitter—receiver pairs located in the same array were forcedly nullified to exclude the values that were not related to the sound propagation in fluid from the analysis. Thus, the matrix contained 64×48 nonzero elements.

4. OBJECTS OF STUDY AND EXPERIMENTAL RESULTS

The above-described tomographic experiments were carried out on two fluid objects. The first was a stationary vortex excited in a homogeneous fluid. To this end, a cell filled with water was placed on the platform of magnetic mixer (IKA-Werke). The mixer rotor, rotating with a speed of 1500 rpm, was located in the central part of the cell (see Fig. 2). The rotor was a disk magnet 5 mm in diameter, inserted in a 13-mm-long plastic tube. In view of the difference in the cell thickness (25 mm) and the width of the active region of piezoelectric elements (10 mm), the rotor was located beyond the main propagation region of the sound fields emitted by arrays. The plastic tube additionally absorbed ultrasound to reduce backscattering.

The experimental results are shown in Fig. 3. Figure 3(a) presents a two-dimensional tomographic reconstruction of the modulus of fluid velocity in gray scale. Figure 3(b) shows the velocity vector field. Note that the velocity is normalized both in the luminous images and in the vector diagram. The image size is 30×30 mm, with allowance for the 10 mm standoff from the cell edges, where the image has distortions related to the proximity to the surface of the antenna arrays. A stable vortex in the form of a helix with maximum velocity of about 5 cm s^{-1} was formed in the cell center. The rotation speed of the central part of the vortex above the rotor was 2.5 rps, i.e., lower than the rotor rotation speed by a factor of 10.

The second object was a combination of two nonmixing fluids, distilled water and silicone oil with a viscosity of 1.5 cSt (Dow Corning), placed in the cell at a temperature of 20°C. The silicone oil density and the speed of sound in this oil are, respectively, $\rho_{\rm oil} = 847$ kg m^{-3} and $c_{\rm oil} = 960$ m s^{-1}. The components filled completely the cell volume in a ratio of about 1:1. The cell was mounted vertically on a vibration machine and subjected to sinusoidal oscillations in the vertical plane. Vibrations at doubled frequencies of standing capillary-gravity waves in this system may lead to instability, and at that in a certain range of excitation vibration amplitudes a steady-state mode can be obtained. The experiments were performed with the following values of vibration amplitude and frequency, corresponding to excitation of steady-state vibrations of the first mode (along the cell width): 10 mm and 3.5 Hz, respectively. The vibrations set at a frequency of 1.75 Hz covered a significant part of the cell volume and had a vertical deviation amplitude $\geqslant$12 mm on its axis. Since the duration of one measurement cycle (80 ms) was comparable with 1/8 vibration period, the second object could be displaced by a significant distance for the observation time. Therefore, the measurements were performed in the so-called sampling mode in this case. Four successive samples of data for each array were recorded in the same phase from different stationary vibration cycles. This approach allowed us to reduce the effective measurement time to 20 ms, which is comparable with 1/32 period and can be considered sufficiently short to exclude any significant changes in the object configuration and velocity for this time interval.

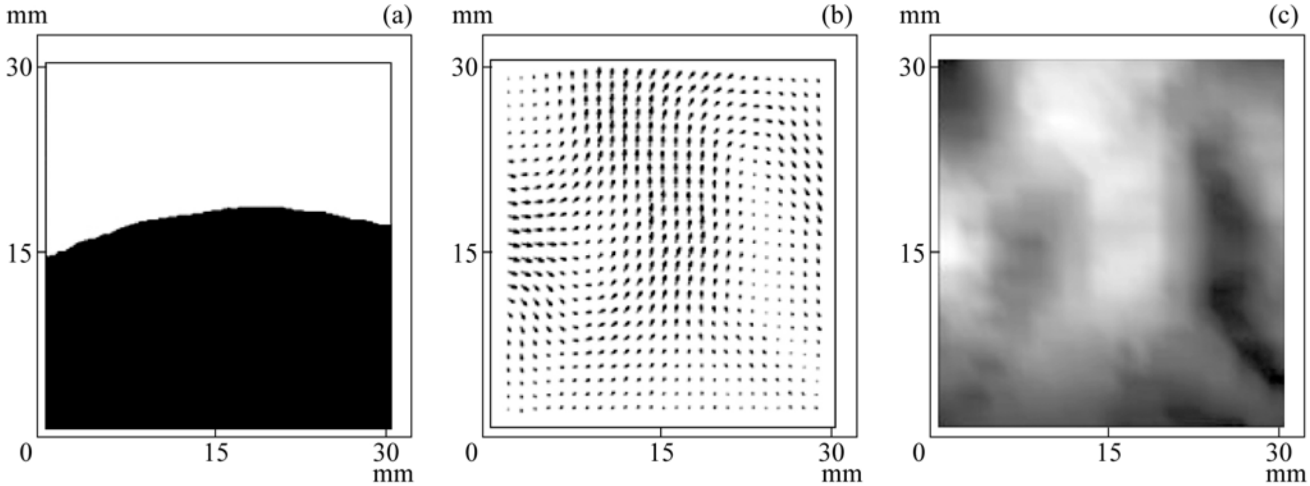

Fig. 4. Reconstruction of distributions recorded at the instant of passage through equilibrium state: (a) scalar field of speeds of sound in the cell, (b) flow velocity vector field, and (c) distribution of flow velocity moduli.

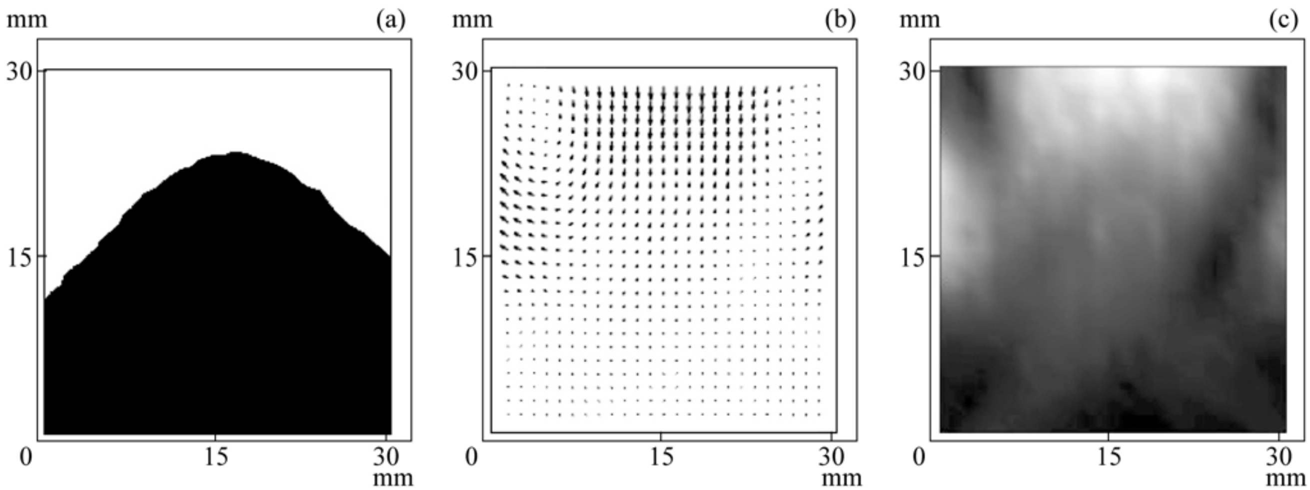

Fig. 5. Reconstruction of distributions recorded at the instant of maximum deviation from equilibrium: (a) scalar field of speeds of sound in the cell, (b) flow velocity vector field, and (c) distribution of flow velocity moduli.

Thus, the second object can be described by a vector field of velocity distribution and a scalar field of speeds of sound (i.e., the distribution of the fluids). Figure 4 shows reconstructed characteristics of the measured mode of standing wave in the vibration phase when the wave passes near the equilibrium position.

Figure 4(a) shows a scalar field of speeds of sound in the cell (in shades of black and white), corresponding to the wave location at the instant of detection. Figure 4(b) presents the velocity vector field, and Fig. 4(c) shows the distribution of the flow velocity modulus (in gray scale). These results indicate formation of two circular flows in the cell, the field of which is characteristic of the distribution of vibrational velocity of a gravitational wave in a fluid.

Figure 5 shows similar distributions obtained near the maximum-deviation phase. A fluids distribution image in shades of black and white is shown in Fig. 5(a). The velocity vector field (Fig. 5(b)) indicates that the distribution was recorded at the instant when the deviation peak began to fall off and the backward fluid flow (a bright spot in Fig. 5(c)) started forming.

The results of this study demonstrated a possibility of reconstructing simultaneously ultrasonic tomographic images of the scalar distribution of the composition of a two-component system of nonmixing fluids and the vector field of flows in these fluids. Thus, ultrasonic tomographic systems can successfully be used to visualize flows of dissimilar fluids.

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